Glossary of Z notation

Names

- \(a, b\) identifiers
- \(d, e\) declarations (e.g., \(a : A; b : \ldots : B\ldots\))
- \(f, g\) functions
- \(m, n\) numbers
- \(p, q\) predicates
- \(s, t\) sequences
- \(x, y\) expressions
- \(A, B\) sets
- \(C, D\) bags
- \(Q, R\) relations
- \(S, T\) schemas
- \(X\) schema text (e.g., \(d, d\{p\} \text{ or } S\))

Definitions

- \(a \equiv x\) Abbreviated definition
- \(a :: b\ldots\) Data type definition (or \(a :: b(\{x\})\ldots\))
- \([a]\) Introduction of a given set (or \([a, \ldots]\))
- \(-a\) Prefix operator
- \(-a\) Postfix operator
- \(-a\) Infix operator

Logic

- \(\text{true}\) Logical true constant
- \(\text{false}\) Logical false constant
- \(\neg p\) Logical negation
- \(p \land q\) Logical conjunction
- \(p \lor q\) Logical disjunction
- \(p \Rightarrow q\) Logical implication (\(\neg p \lor q\))
- \(p \Leftrightarrow q\) Logical equivalence (\(p \Rightarrow q \land q \Rightarrow p\))
- \(\forall X \bullet q\) Universal quantification
- \(\exists X \bullet q\) Existential quantification
- \(\exists_1 X \bullet q\) Unique existential quantification
- let \(a \equiv x; \ldots \bullet p\) Local definition

Sets and expressions

- \(x = y\) Equality of expressions
- \(x \neq y\) Inequality (\(\neg (x = y)\))
- \(x \in A\) Set membership
- \(x \notin A\) Non-membership (\(\neg (x \in A)\))
- \(\emptyset\) Empty set
- \(A \subseteq B\) Set inclusion
- \(A \subset B\) Strict set inclusion (\(A \subseteq B \land A \neq B\))
- \(\{x, y, \ldots\}\) Set of elements
- \(\{X \bullet x\}\) Set comprehension
- \(\lambda X \bullet x\) Lambda-expression – function
- \(\mu X \bullet x\) Mu-expression – unique value

Relations

- \(A \leftrightarrow B\) Relation (\(P(A \times B)\))
- \(a \mapsto b\) Maplet (\(\{a, (b, b), \ldots\}\))
- \(\text{dom } R\) Domain of a relation
- \(\text{ran } R\) Range of a relation
- \(\text{id } A\) Identity relation
- \(Q \\ R\) Forward relational composition
- \(Q \circ R\) Backward relational composition (\(R^{-1} Q\))
- \(A \preceq R\) Domain restriction
- \(A \ll R\) Domain anti-restriction
- \(A \gg R\) Range restriction
- \(A \ggg R\) Range anti-restriction
- \(R[\{a\}]\) Relational image
- \(\text{iter } n R\) Relation composed \(n\) times
- \(R^n\) Same as \(\text{iter } n R\)
- \(R^\sim\) Inverse of relation (\(R^{-1}\))
- \(R^*\) Reflexive-transitive closure
- \(\text{co } R\) Irreflexive-transitive closure
- \(Q \oplus R\) Relational overriding (\(\{\text{dom } R \ll Q\} \cup R\))
- \(a \text{ co } R b\) Infix relation

Functions

- \(A \rightarrow B\) Partial functions
- \(A \rightarrow B\) Total functions
- \(A \rightarrow B\) Partial injections
- \(A \rightarrow B\) Total injections
- \(A \rightarrow B\) Partial surjections
- \(A \rightarrow B\) Total surjections
- \(A \rightarrow B\) Bijective functions
- \(A \rightarrow B\) Finite partial functions
- \(A \rightarrow B\) Finite partial injections
- \(f \bullet x\) Function application (or \(f(x)\))
Numbers

\[ \mathbb{Z} \] Set of integers
\[ \mathbb{N} \] Set of natural numbers \{0, 1, 2, \ldots\}
\[ \mathbb{N}_1 \] Set of non-zero natural numbers (\( \mathbb{N} \setminus \{0\} \))
\[ m + n \] Addition
\[ m - n \] Subtraction
\[ m \times n \] Multiplication
\[ m \div n \] Division
\[ m \mod n \] Modulo arithmetic
\[ m \leq n \] Less than or equal
\[ m < n \] Less than
\[ m \geq n \] Greater than or equal
\[ m > n \] Greater than
\[ \text{succ} n \] Successor function \( \{0 \mapsto 1, 1 \mapsto 2, \ldots\} \)
\[ m \ldots n \] Number range
\[ \min A \] Minimum of a set of numbers
\[ \max A \] Maximum of a set of numbers

Sequences

\( \text{seq} A \) Set of finite sequences
\( \text{seq}_1 A \) Set of non-empty finite sequences
\( \text{i seq} A \) Set of finite injective sequences
\( \langle \rangle \) Empty sequence
\( \langle x, y, \ldots \rangle \) Sequence \( \{1 \mapsto x, 2 \mapsto y, \ldots\} \)
\( \langle t \rangle \) Sequence concatenation
\( ^{\sim} / s \) Distributed sequence concatenation
\( \text{head} s \) First element of sequence \( (s(1)) \)
\( \text{tail} s \) All but the head element of a sequence
\( \text{last} s \) Last element of sequence \( (s(#s)) \)
\( \text{front} s \) All but the last element of a sequence
\( \text{rev} s \) Reverse a sequence
\( \text{squash} f \) Compact a function to a sequence
\( A \mid s \) Sequence extraction \( (\text{squash}(A \triangleleft s)) \)
\( s \mid A \) Sequence filtering \( (\text{squash}(s \triangleright A)) \)
\( s \text{ prefix} t \) Sequence prefix relation \( (s \sim v = t) \)
\( s \text{ suffix} t \) Sequence suffix relation \( (u \sim s = t) \)
\( s \text{ in} t \) Sequence segment relation \( (u \sim s \sim v = t) \)
\( \text{disjoint} A \) Disjointness of an indexed family of sets
\( A \text{ partition} B \) Partition an indexed family of sets

Bags

\( \text{bag} A \) Set of bags or multisets \( (A \rightarrow \mathbb{N}_1) \)
\( \lbrack \rbrack \) Empty bag
\( \lbrack x, y, \ldots \rbrack \) Bag \( \{x \mapsto 1, y \mapsto 1, \ldots\} \)
\( \text{count} C x \) Multiplicity of an element in a bag
\( C \preceq x \) Same as \( \text{count} C x \)
\( n \odot C \) Bag scaling of multiplicity
\( x \in C \) Bag membership
\( C \subseteq D \) Sub-bag relation
\( C \sqcup D \) Bag union

\( C \sqcup D \) Bag difference
\( i \in m s s \) Bag of elements in a sequence

Schema notation

Vertical schema.

A new line denotes ‘;’ and ‘\&’. The schema name and predicate part are optional. The schema may subsequently be referenced by name in the document.

Axiomatic definition.

The definitions may be non-unique. The predicate part is optional. The definitions apply globally in the document.

Generic definition.

The generic parameters are optional. The definitions must be unique. The definitions apply globally in the document.

\[ S \triangle [X] \] Horizontal schema
\[ [T;\ldots;] \] Schema inclusion
\( z,a \) Component selection (given \( z : S \))
\( \theta S \) Tuple of components
\( \neg S \) Schema negation
\( \text{pre} S \) Schema precondition
\( S \land T \) Schema conjunction
\( S \lor T \) Schema disjunction
\( S \Rightarrow T \) Schema implication
\( S \Leftarrow T \) Schema equivalence
\( S \setminus (a,\ldots) \) Hiding of component(s)
\( S \mid T \) Projection of components
\( S \circ T \) Schema composition \( (S \text{ then } T) \)
\( S \gg T \) Schema piping \( (S \text{ outputs to } T \text{ inputs}) \)
\( S[a/b,\ldots] \) Schema component renaming \( (b \text{ becomes } a, \text{ etc.}) \)
\( \forall X \cdot S \) Schema universal quantification
\( \exists X \cdot S \) Schema existential quantification
\( \exists_1 X \cdot S \) Schema unique existential quantification

Conventions

\( a? \) Input to an operation
\( a! \) Output from an operation
\( a \) State component before an operation
\( a' \) State component after an operation
\( S \) State schema before an operation
\( S' \) State schema after an operation
\( \Delta S \) Change of state \( (\text{normally } S \land S') \)
\( \Xi S \) No change of state \( (\text{normally } [S \land S'] \theta S = \theta S') \)

Jonathan P. Bowen
Oxford University Computing Laboratory
Wolfson Building, Parks Road, OXFORD OX1 3QD, UK
Email: Jonathan.Bowen@comlab.ox.ac.uk