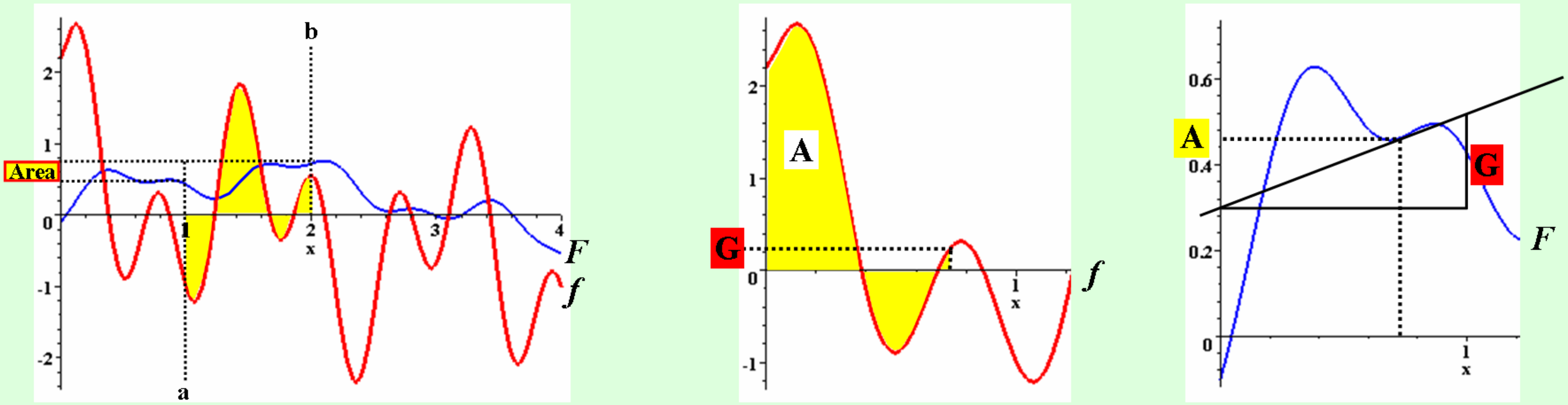
THEOREM OF THE DAY

The Fundamental Theorem of the Calculus

Part I: If f is continuous on the closed interval $[a, b]$ and $F(x)$ is a function for which $\frac{dF}{dx} = f(x)$ in that interval (F is an antiderivative of f), then $\int_a^b f(x)dx = F(b) - F(a)$.

Part II: An antiderivative for f exists: if f is continuous on an open interval around zero, and $F(x) = \int_0^x f(x)dx$, then $\frac{dF}{dx} = f(x)$, for every point in that interval.



Part I **Part II**

Part I: No matter how complicated the continuous function f in the interval $[a, b]$, if we can find an antiderivative F, then calculating the shaded area is just a simple subtraction: $F(b) - F(a)$. The area under curve $f(x)$ from $x = a$ to $x = b$ can be identified with the *definite integral* $\int_a^b f(x)dx$ (it does no harm to think of $\int_a^b dx$ as a rather unusual notation for area).

Part II: And we can find an antiderivative F by measuring the area under f at every point away from zero. Unfortunately this will not normally give us a nice closed formula whereby area might be measured so neatly using Part I. Nevertheless, the duality between f and F that maps values G of the function f to gradients G on the curve F (values of dF/dx); and areas A under curve f (values of $\int f dx$) to values A of the function F, is one of the most fundamental in mathematics.

The Fundamental Theorem of the Calculus was discovered by Isaac Barrow (1630 – 1677) and was first explicitly stated by Isaac Newton who took over the Lucasian Chair of Mathematics at Cambridge from Barrow in 1669. The calculus was not made fully rigorous until the work of Augustin Louis Cauchy and others almost two hundred years later.

Web link: ugrad.math.ubc.ca/coursedoc/math101/notes/integration/ftc.html

Further reading: *Calculus Made Easy* by Silvanus P. Thompson and Martin Gardner (Editor), Palgrave Macmillan, 1999.