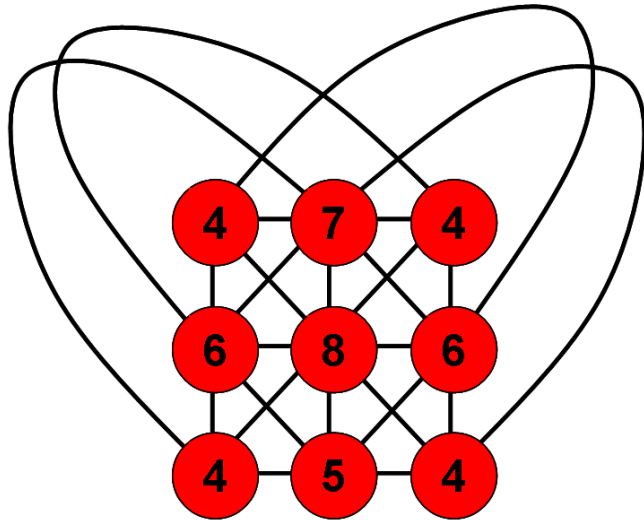


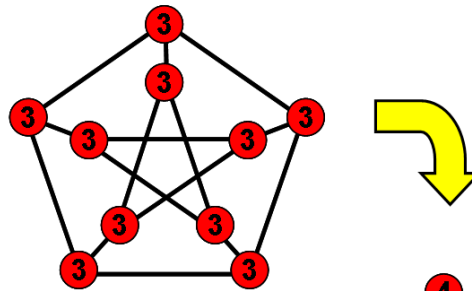


THEOREM OF THE DAY

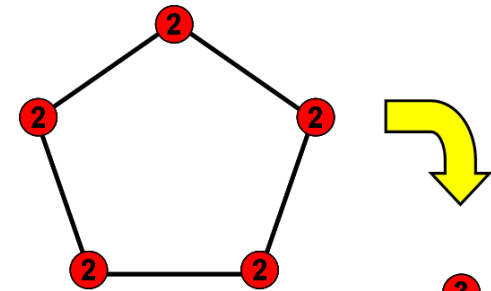
Graph Colouring by Vertex Degree (a Theorem Under Construction!) Call a graph, G , degree colourable if no two adjacent vertices have the same degree. If, by adjusting edge multiplicities in G so that no edge multiplicity exceeds k , we can create a degree colourable graph, then say that G is k -degree colourable. Now there exists a positive integer K , not dependent on the size of G , such that any connected graph on at least three vertices is K -degree colourable.



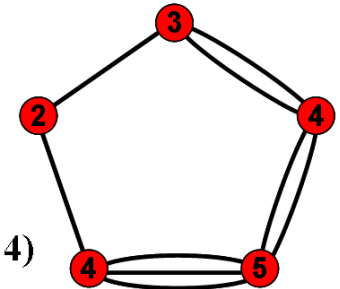
A graph based on the affine plane of order 3: $k = 1$



The Petersen graph: $k = 2$



The n -cycle: $k = 3$ unless $n \equiv 0 \pmod{4}$



The degree of a vertex is the number of edges incident with that vertex. Above left, a graph based on (a representation of) the affine plane of order 3 is already properly coloured by its degrees. In the middle, the so-called Petersen graph is clearly not, since it is 3-regular (every vertex has degree 3) but doubling five suitably chosen edges shows that it is 2-degree colourable. Finally, doubling edges in the n -cycle is not sufficient, unless n is a multiple of 4, but degree colourability is 3: tripling edges is always enough.

Construction notes:



- 2002: Karoński, Łuczak and Thomason conjecture that K exists and equals 3. Prove it for 3-colourable graphs.
- 2004: Addario-Berry, Dalal, McDiarmid, Reed and Thomason prove that K exists and that $K=30$ is sufficient.
- 2004: Addario-Berry, Dalal and Reed prove that $K=16$ is sufficient.
- 2008: Tao Wang and Qinglin Yu reduce K to 13.
- 2009: Maciej Kalkowski, Michał Karoński and Florian Pfender reduce K to 5.

Web link: www.math.uni-rostock.de/~pfender/papers/22.pdf

Further reading: *Graph Theory (2nd Edition)* by Reinhard Diestel, Springer New York, 2000.

