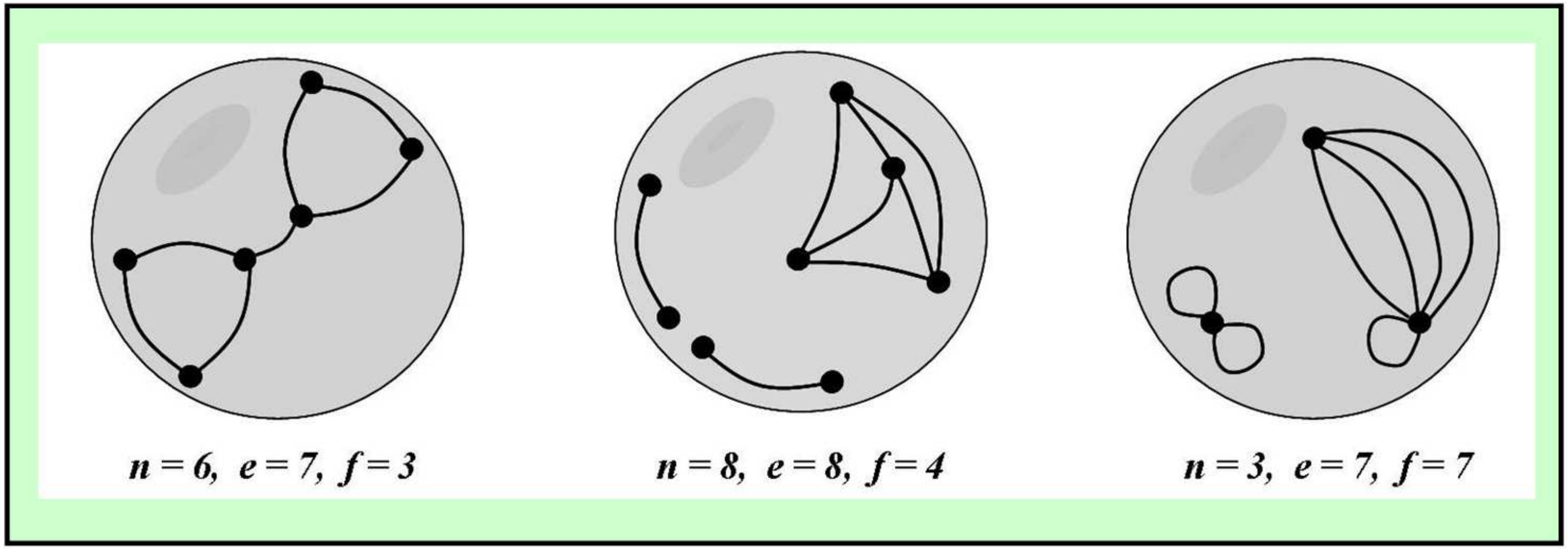
THEOREM OF THE DAY

Euler's Formula *For a connected plane graph G with n vertices, e edges and f faces,*

$$n - e + f = 2.$$



Plane graphs are those which have been drawn on a plane or sphere with edges meeting only at vertices. Only for the first of these 3 spheres does the formula appear to work ($6 - 7 + 3 = 2$) but the graph drawn on the second sphere is not connected, having 3 components, while that on the third has 2 components. We can extend Euler's formula: $n - e + f = c + 1$, where c is the number of connected components. If we take all three spheres together we get a graph with six components illustrating a further extension of the formula: $n - e + f = s + c$ where s is the number of spheres. This is one way of explaining where the number 2 in Euler's original formula comes from: 1 connected component drawn on 1 sphere.

Euler, in 1750, seems to have been the first to observe that $e - n + f = 2$ holds for 3-dimensional polyhedra such as the cube or dodecahedron, even though such platonic solids were intensively studied by the ancient Greeks. Euler did not prove the formula correctly, this being first done by Legendre in 1794. Since then it has been generalised continuously as well as finding applications in all kinds of mathematics, notably the proof of the four colour theorem.

Web link: www.ams.org/featurecolumn/archive/eulers-formula.html

Further reading: *Proofs and Refutations. The Logic of Mathematical Discovery* by Imre Lakatos, edited by Worrall and Zahar, Cambridge University Press, 1976.