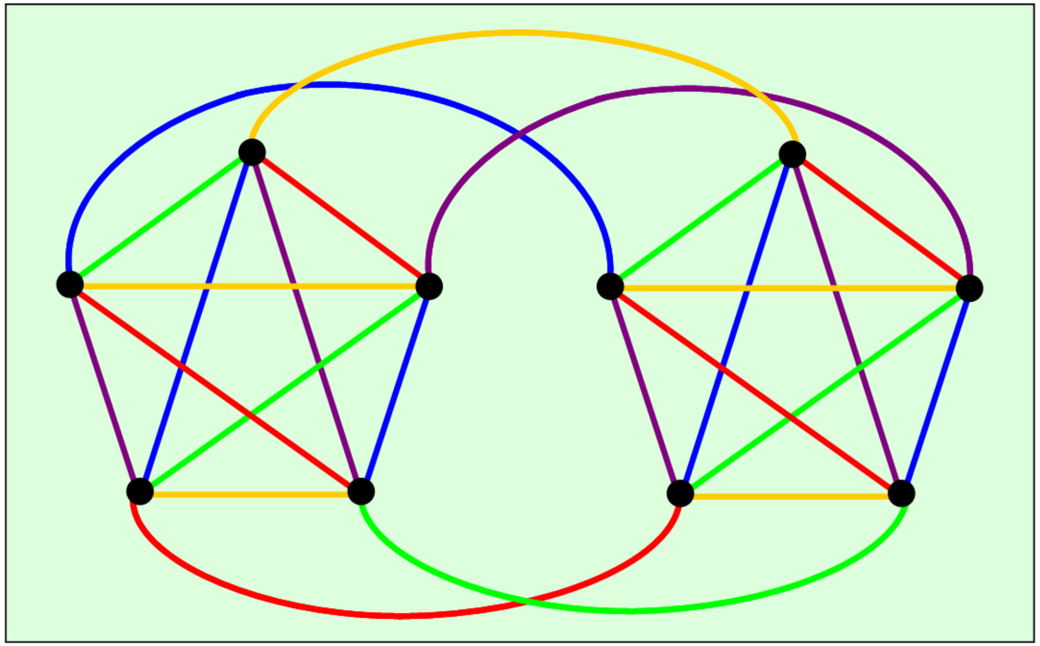
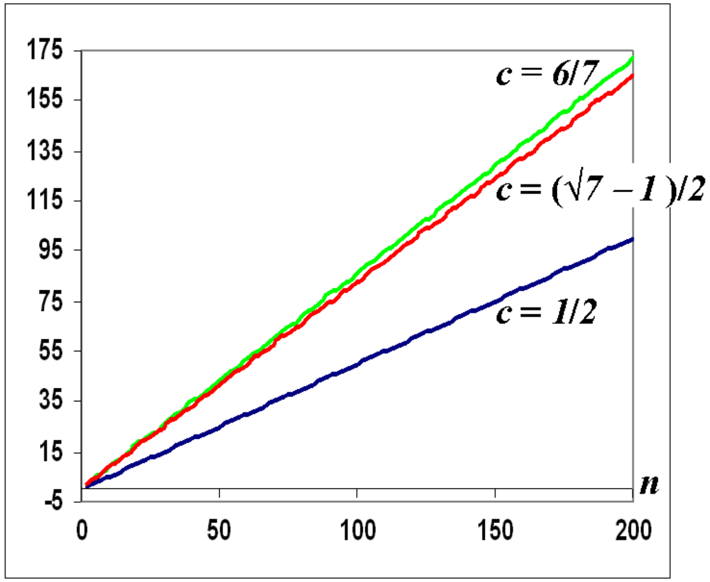
# THEOREM OF THE DAY

**1-Factorisation of Regular Graphs (a Theorem Under Construction!)** *There exists a constant, c, such that all simple graphs G of even order, n, that are d-regular with $d \geq cn$, have a 1-factorisation.*

Colour-free version



**A 1-factorisation of $K_2 \times K_5$ ($n = 10$, $d = n/2$)**




**Comparison of *cn* with increasing *n***

The graph $K_2 \times K_t$ (2 copies of $K_t$ with edges joining corresponding vertices) has a 1-factorisation for all $t \geq 1$ and is $d$-regular with $d = cn$, $c = 1/2$ (that is, every vertex is incident with $n/2$ edges). This is illustrated, above left, for $t = 5$ (with, what is more, a *perfect* 1-factorisation: any pair of edge-colours produces a Hamiltonian circuit in the graph); but the ultimate goal of $c = 1/2$ is well below what has been achieved, so far, for general $d$-regular graphs, as the right-hand chart shows.

**Construction notes:** 1950s: Known that $K_{2n}$ factorises. Dirac, Nash-Williams (and others?) conjecture that c exists and equals 1/2.
1985: Breakthrough! Amanda Chetwynd and Anthony Hilton prove existence of c by showing c≤6/7 (=24/28).
1985: R. Häggkvist proves ∀ε, can take c=1/2+ε for large enough n ('97: published independently, Perković & Reed).
1989: Chetwynd and Hilton achieve c=(√7-1)/2≈24/29 (as do Niessen and Volkmann independently, 1990).
2004: Hilton's student David Cariolaro achieves c=(√57-3)/6≈22/29, except for 2 special classes of d-regular graphs.

**Web link:** www.math.sinica.edu.tw/post-doctor/cariolaro/webtutte2.pdf

**Further reading:** *Graphs and Digraphs (4th Edition)* by G. Chartrand and L. Lesniak, Chapman & Hall/CRC, 2004.