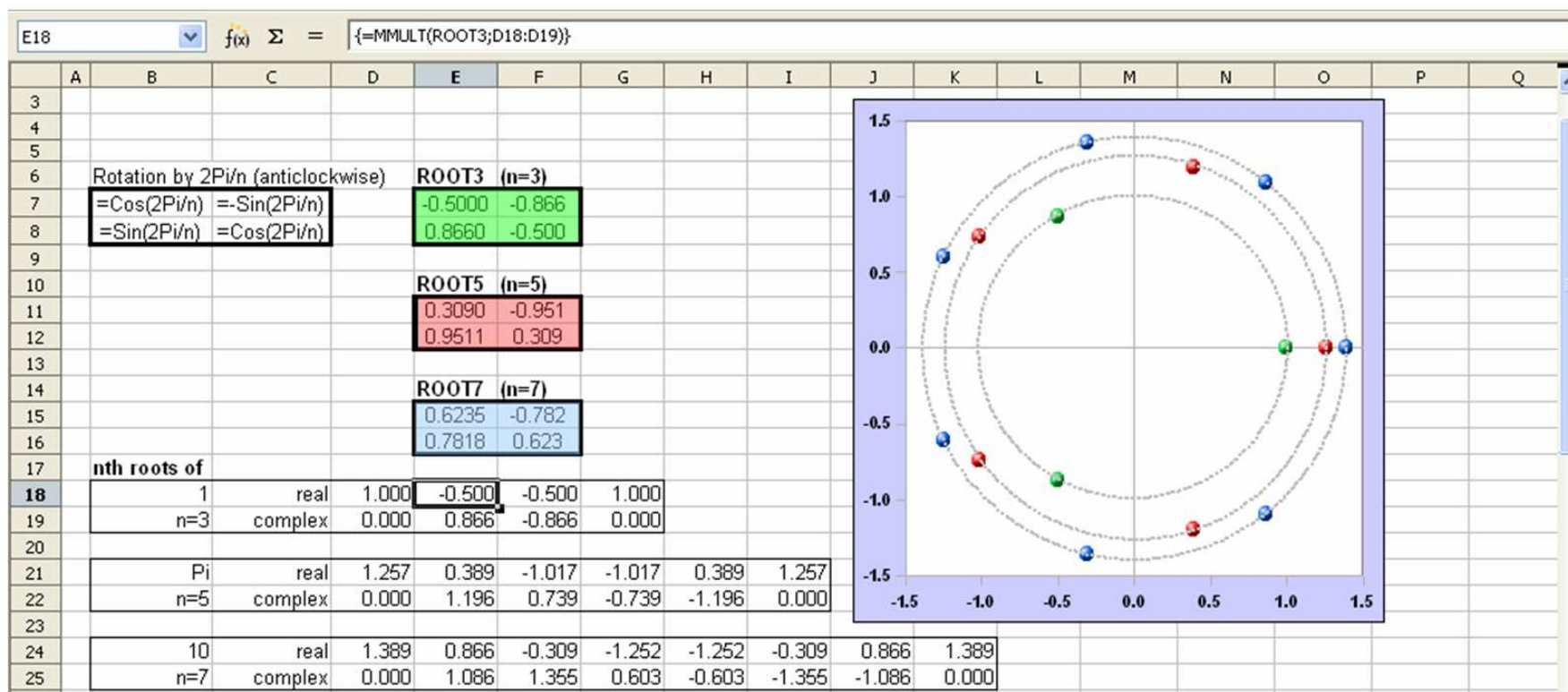


THEOREM OF THE DAY

De Moivre's Theorem *Let θ be an angle and n a positive integer. Then*

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta.$$



Among other things, De Moivre's theorem is perfect for finding n -th roots of real or complex numbers, as shown in the above screenshot from **OpenOffice Calc**. In the plot we think of the horizontal axis as recording the real part and the vertical axis the imaginary part of the complex number $z = a + ib$. This number may be written as $z = r(\cos \theta + i \sin \theta)$ where $r = \sqrt{a^2 + b^2}$ and θ is the angle made with the horizontal axis by the line from $(0, 0)$ to (a, b) : $\cos \theta = a/r$. On our horizontal axis three real numbers are plotted: $\sqrt[3]{1} = 1$, $\sqrt[3]{\pi} \approx 1.257$ and $\sqrt[3]{10} \approx 1.389$. But there are complex n -th roots too! For instance, the complex number z will be cube root of 1 if $z^3 = r^3(\cos \theta + i \sin \theta)^3 = r^3(\cos 3\theta + i \sin 3\theta) = 1$. This will happen whenever 3θ is a multiple of 2π because $\cos 2\pi + i \sin 2\pi = 1 + i \times 0 = 1$. So in our plot we have the point $(1, 0)$ and this point rotated by $2\pi/3$ and by $4\pi/3$ to give the three cube roots of 1. A nice way to rotate a point (x, y) by an angle $2\pi/n$ is to multiply the point by the appropriate *rotation matrix* as shown above left. Note that $n = 2$ gives a rotation of the real square root of a number x by $2\pi/2$, or 180 degrees, to give $-x$.

Abraham de Moivre discovered a version of his formula in 1707 and proposed the more usual version in 1722. Leonhard Euler's famous 1749 identity $e^{i\theta} = \cos \theta + i \sin \theta$ generalised it to real number exponents.

Web link: archimede.mat.ulaval.ca/pages/genest/publi/StatSci-2007.pdf

Further reading: *Trigonometric Delights* by Eil Maor, Princeton University Press, 2002

