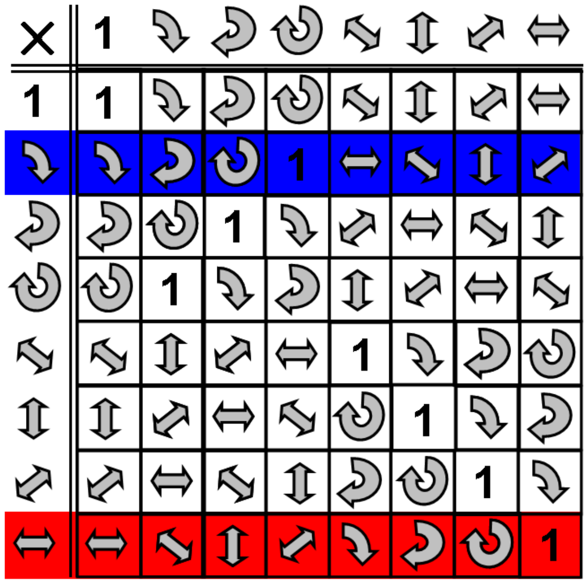
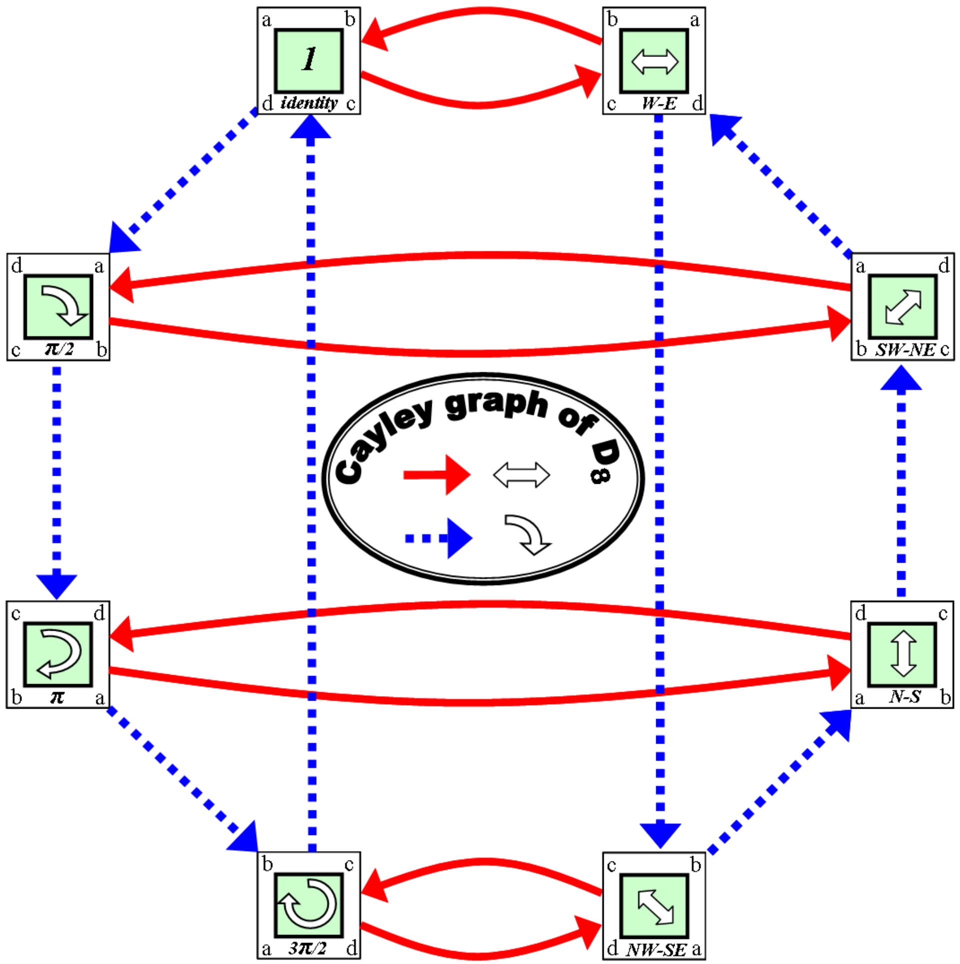
THEOREM OF THE DAY

Cayley's Theorem *A finite group of order n is isomorphic to a subgroup of the symmetric group on n points.*



A D_8 times-table

The symmetric group S_8 consists of all permutations of 8 objects. Each row of the multiplication table for D_8, the symmetries of the square, clearly constitutes a permutation among the 8 possible symmetries. The rows belonging to two symmetries which, between them, generate all others produce a so-called *Cayley graph* (shown on the right). The permutations are then seen graphically: rotation by $\pi/2$ permutes D_8 in two 4-cycles; the West–East flip in four 2-cycles.



Although Cayley's Theorem is classic textbook stuff, I have recorded no less an authority than Peter M. Neumann referring to it as "an important theorem in the nineteenth century; a pretty feeble theorem in the twentieth; an even more feeble theorem now." Arthur Cayley published the theorem in 1854 in one of the first ever papers on group theory. At that time, Neumann said, it would have been a kind of 'comfort blanket' for mathematicians unfamiliar with the abstract idea of a group.

Web link: http://www-history.mcs.st-and.ac.uk/~edmund/lnotes/node14.html

Further reading: *Groups and Geometry* by Peter M. Neumann, Gabrielle A. Stoy and Edward C. Thomson, OUP, 1994.