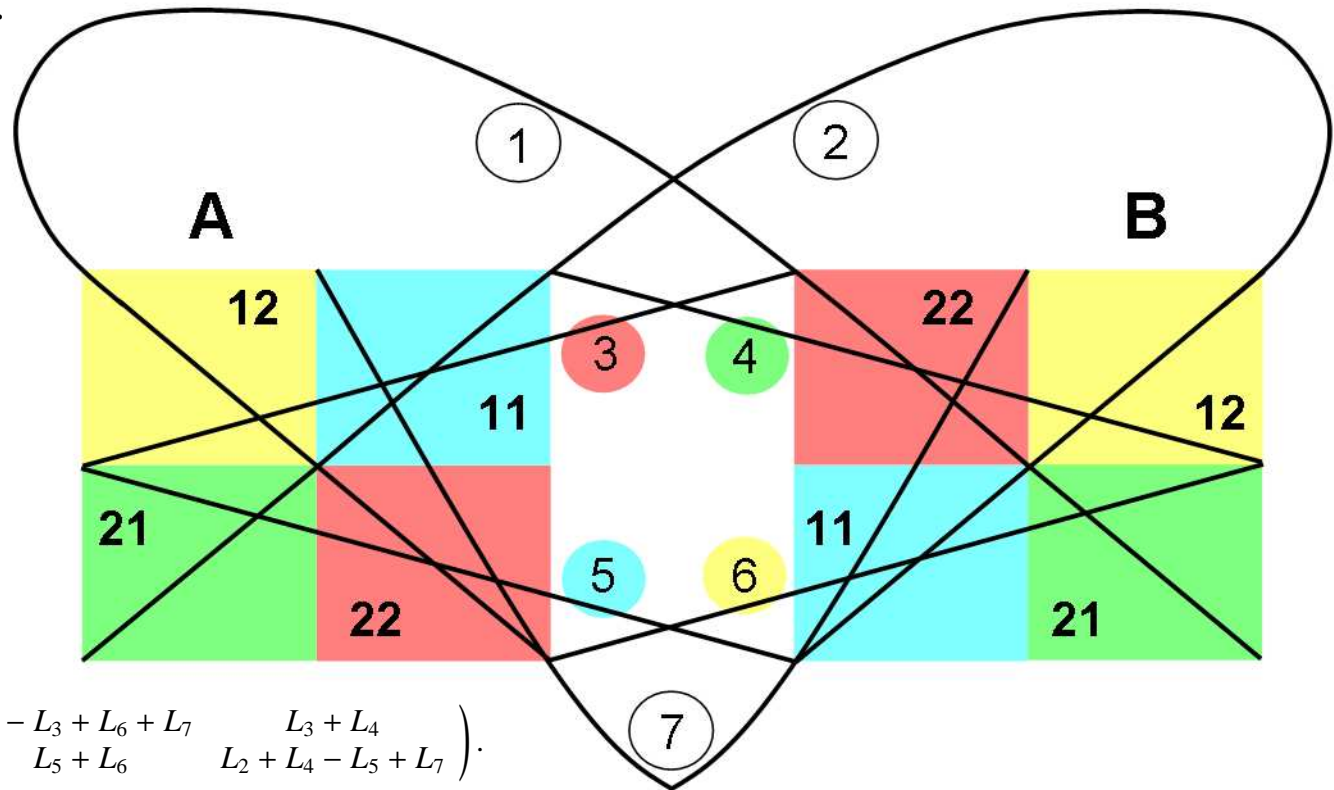


# THEOREM OF THE DAY

**Strassen's Matrix Theorem (a Theorem Under Construction!)** *Two  $n \times n$  matrices can be multiplied in fewer than  $n^3$  (multiplication) steps.*

## Instructions for using this diagram

1. Entries of matrices  $A$  and  $B$  are arranged as shown. Now seven products are formed as follows:
2. Entries along curved lines ( $L_1, L_2$  and  $L_7$ , passing four entries) and straight lines ( $L_3, L_4, L_5$  and  $L_6$ , passing three) are added *within each matrix* and the sums are multiplied *between the matrices*. Line  $L_7$  gives  $(a_{11} + a_{22}) \times (b_{11} + b_{22})$ , for example. *But ...*
3. ... where green (21) and blue (11) appear consecutively, the blue entry is subtracted rather than added; thus  $L_6$  gives  $a_{22}(b_{21} - b_{11})$ ; and ...
4. ... a red (22) consecutively with yellow (12) is likewise subtracted:  $L_1$  gives  $(a_{12} - a_{22})(b_{21} + b_{22})$ .
5. Now  $AB = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix} = \begin{pmatrix} L_1 - L_3 + L_6 + L_7 & L_3 + L_4 \\ L_5 + L_6 & L_2 + L_4 - L_5 + L_7 \end{pmatrix}$ .
6. For larger matrices (padded out to size  $2^k \times 2^k$  as necessary) partition into square blocks and proceed as above.



## Construction notes:

- 1969: Volker Strassen amazes the world by multiplying  $n \times n$  matrices in  $O(n^{2.81})$  (multiplication) steps.
- 1970s & 80s: Strassen's result leads to a succession of lower values for the exponent, which becomes known as  $\omega$ .
- 1987: Don Coppersmith and Shmuel Winograd achieve best value for  $\omega$  to date with  $O(n^{2.38})$  algorithm.
- 2003: Chris Umans and Henry Cohn discover group theoretic formulation of Strassen's approach.
- 2005: With Robert Kleinberg and Balazs Szegedy they reproduce Coppersmith and Winograd group theoretically and give two conjectures whose proof will reduce  $\omega$  to its theoretical minimum value of 2. Meanwhile, Strassen's original method remains the fastest practical way to multiply matrices larger than around  $30 \times 30$ .



Web link: [www.siam.org/pdf/news/174.pdf](http://www.siam.org/pdf/news/174.pdf)

Further reading: *Algorithms and Complexity, 2nd edition* by Herbert S. Wilf, A K Peters, 2003.

Theorem of the Day is maintained by Robin Whitty at [www.theoremoftheday.org](http://www.theoremoftheday.org)

