THEOREM OF THE DAY

Germain's Theorem *If p is an odd prime such that $2p+1$ is also prime and if x, y and z are integers none of which is divisible by p then $x^p + y^p \neq z^p$. Such x, y and z cannot therefore be counterexamples to Fermat's Last Theorem for exponent p.*

1	2	3	4	?	6	?	8	9	10
?	12	?	14	15	16	?	18	?	20
21	22	?	24	25	26	27	28	?	30
?	32	33	34	35	36	?	38	39	40
?	42	?	44	45	46	?	48	49	50
51	52	?	54	55	56	57	58	?	60
?	62	63	64	65	66	?	68	69	70
?	72	?	74	75	76	77	78	?	80
81	82	?	84	85	86	87	88	?	90
91	92	93	94	95	96	?	98	99	100

1	2	3	4	5	6	?	8	9	10
11	12	?	14	15	16	?	18	?	20
21	22	23	24	25	26	27	28	29	30
?	32	33	34	35	36	?	38	39	40
41	42	?	44	45	46	?	48	49	50
51	52	53	54	55	56	57	58	?	60
?	62	63	64	65	66	?	68	69	70
?	72	?	74	75	76	77	78	?	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	?	98	99	100

The state of play regarding Fermat's Last Theorem (FLT) in 1820: the non-solvability of $x^n + y^n = z^n$ by integers for $n > 2$, had been proved securely only for $n = 4$. This case was due to Fermat himself and ruled out any multiple of 4 as well since if, for example, $x^8 + y^8 = z^8$ then $X = x^2, Y = y^2$ and $Z = z^2$ would be integers solving $X^4 + Y^4 = Z^4$. Any other n greater than 4 must contain an odd prime factor and, by a similar argument, this reduces the proof of FLT to $n =$ an odd prime. These are the question marks, shown in the left-hand grid, among the values of n up to 100. In the early 1820s Sophie Germain dramatically changed this picture by providing the first real line on FLT since Fermat's death in 1665. Her work showed that it could be solved in two stages:

Case I: eliminate solutions x, y and z with none being a multiple of n;
Case II: eliminate solutions having one multiple of n,

(two multiples implies all three of x, y and z divide by n and this factor may be removed, reducing back to Case I or II). Germain's Theorem is a powerful condition for Case I to apply, as illustrated by the green squares in the right-hand grid. In fact, her full theorem is even more powerful than what is stated above, whereby she turned all the red squares green, the seventy-year old Adrien-Marie Legendre, with whom she corresponded, continuing this up to $n = 197$. For all these values, FLT had been reduced to Case II.

Sophie Germain (1776 – 1831), who also did pioneering work in the mathematics of elasticity, was able to correspond on equal terms with Gauss and Legendre by disguising her sex behind the pseudonym Monsieur LeBlanc. 'Germain primes' and their application remain a topic of research today, despite the eventual ascent of the FLT mountain by a different face.

Web link: www.agnesscott.edu/lriddle/women/germain-FLT/SGandFLT.htm

Further reading: *Mathematical Expeditions: Chronicles by the Explorers* by R.C. Laubenbacher and D. Pengelley, Springer New York, 2000.