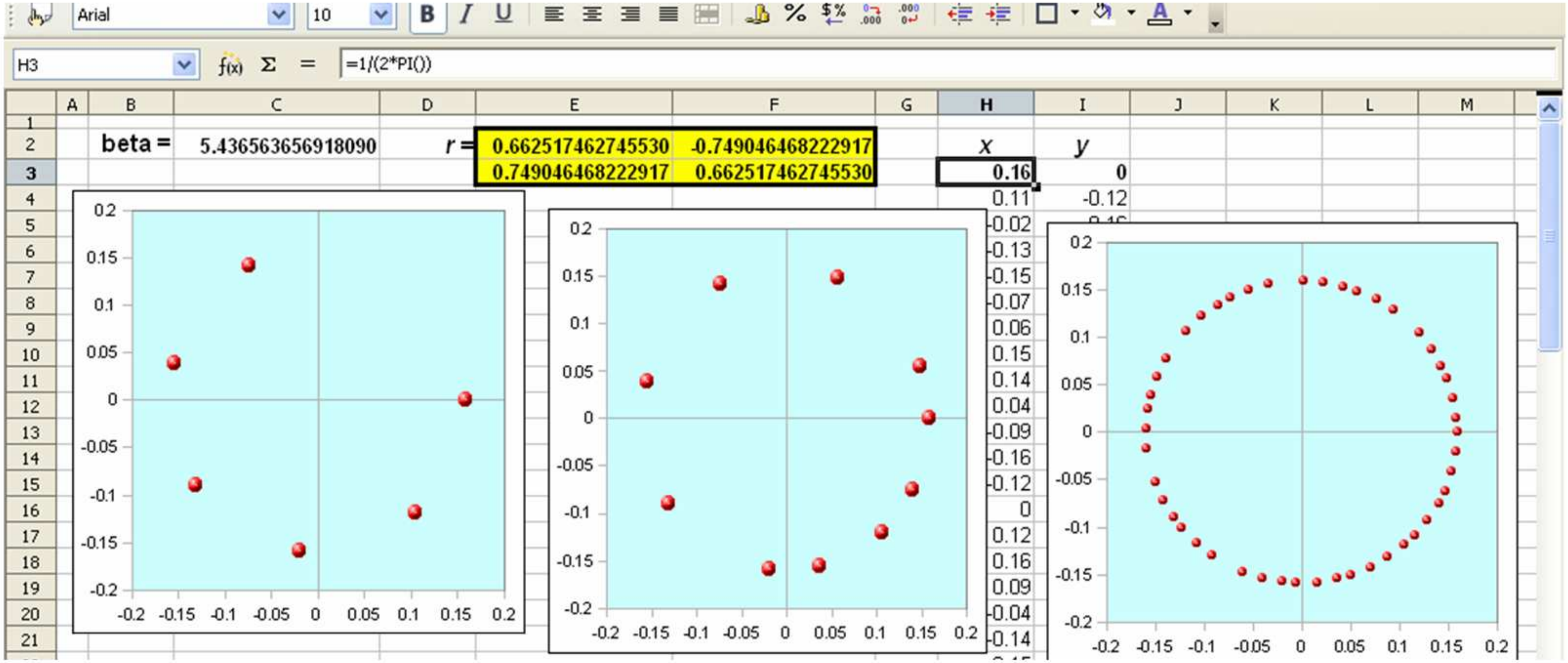
THEOREM OF THE DAY

The Three-Distance Theorem *Let α be an irrational number with $0 < \alpha < 1$ and let n be a positive integer. Then the set of lengths $\{i\alpha \,|\, 0 \leq i \leq n\}$, measured around the circle of circumference 1, partitions the circle into $n + 1$ intervals, whose lengths take at most three values.*



$\alpha = e/\pi, n = 5$ $\alpha = e/\pi, n = 9$ $\alpha = e/\pi, n = 46$

Amazingly, it is unknown whether the ratio $\alpha = e/\pi$ ($\approx 45/52$) is irrational. If it is, then this value of α must obey the Three-Distance Theorem, and this is investigated here using a standard spreadsheet package (in this case Calc by OpenOffice). Starting with point $x = 1/2\pi, y = 0$, in cells H3, I3, we multiply repeatedly by the matrix $r = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}$, $\beta = 2e$, which rotates (x, y) by β radians, anticlockwise around a circle of radius $r = 1/2\pi$, circumference 1, centred on the origin. The arc-length subtended by angle β is $r\beta = 2e/2\pi = \alpha$, so this gives us the partition of the unit circle specified by the theorem. The evidence above does not, so far as it goes, disprove the irrationality of e/π.

Also known as the 'Three-Gap Theorem', this belongs to the field of Diophantine approximation. It was conjectured by the Polish mathematician Hugo Steinhaus (1887–1972) and proved in 1957 by Vera Sós (pronounced 'show-sh').

Web link: www.theoremoftheday.org/Docs/3dAlessandriBerthe.pdf (fine survey by Pascal Alessandri and Valérie Berthé; see section 3).

Further reading: *An Invitation to Modern Number Theory* by S. J. Miller and R. Takloo-Bighash, Princeton University Press, 2006.