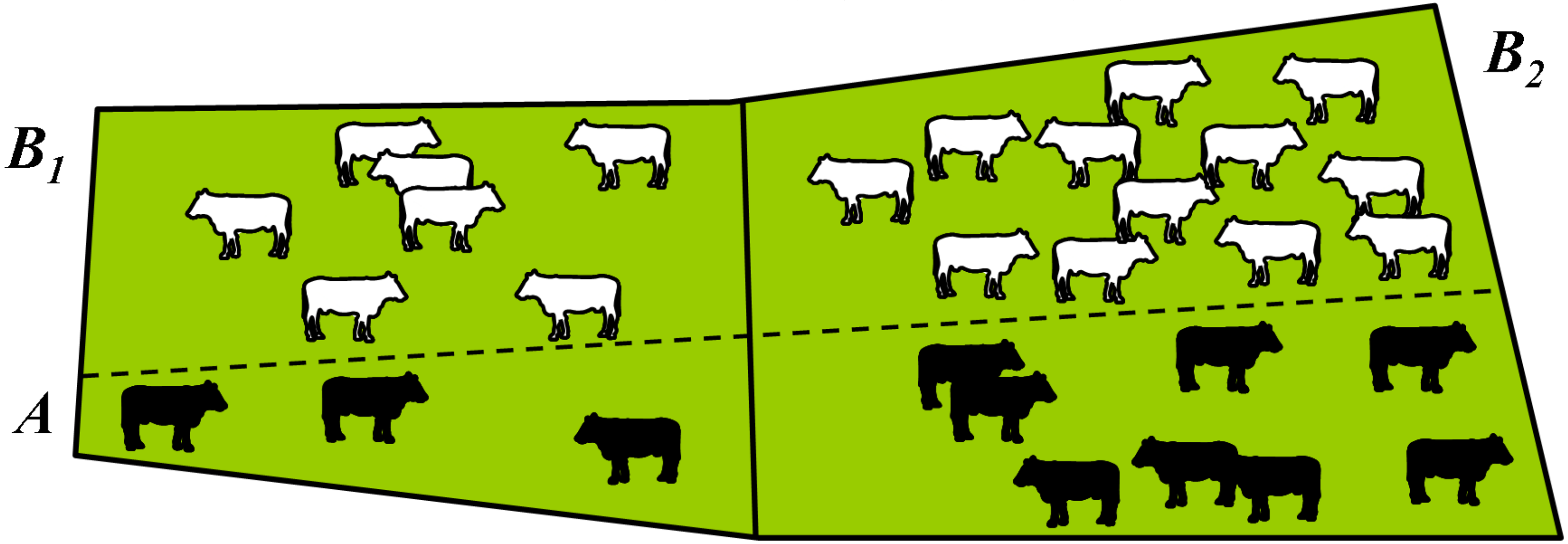
# THEOREM OF THE DAY

**Bayes' Theorem** *Suppose a sample space $S$ is partitioned into two non-empty parts $B_1$ and $B_2$. Then the conditional probability that a point in $S$ satisfying some property $A$ will also lie in $B_1$, is given by*

$$\mathbb{P}(B_1|A) = \frac{\mathbb{P}(A|B_1)\mathbb{P}(B_1)}{\mathbb{P}(A|B_1)\mathbb{P}(B_1) + \mathbb{P}(A|B_2)\mathbb{P}(B_2)}.$$




Thirty cows are grazing, as shown, in two fields, imaginatively called $B_1$ and $B_2$. The black cows have conveniently migrated to the bottom of each field and form a subset $A$ of the cows. We imagine picking a cow uniformly at random. The probability of picking a cow in field $B_1$, $\mathbb{P}(B_1)$, is $10/30 = 1/3$ and $\mathbb{P}(B_2) = 20/30 = 2/3$. The probability of picking a black cow, *given* that you are in field $B_1$, is $\mathbb{P}(A|B_1) = 3/10$, since 3 of the 10 cows in $B_1$ belong in $A$; similarly $\mathbb{P}(A|B_2) = 8/20 = 2/5$. Bayes' theorem tells us that $\mathbb{P}(B_1|A) = \frac{3}{10}.\frac{1}{3}/(\frac{3}{10}.\frac{1}{3} + \frac{2}{5}.\frac{2}{3}) = \frac{3}{11}$, which is clearly correct, since 3 out of the 11 black cows are found in field $B_1$.

The theorem extends to a greater number of regions, $B_1, B_2, B_3, \ldots$, by extending the sum in the denominator in the obvious way; and to a single region, $B_1$, by replacing the denominator by $\mathbb{P}(A)$. The Rev. Thomas Bayes was one of the first to write about conditional probability. His work was published after his death in 1761 but lay for a long time forgotten. His theorem has eventually come to be used controversially to convert possibly subjective assessments of *prior* probability $\mathbb{P}(B_1)$ into a *posterior* probability $\mathbb{P}(B_1|A)$. This so-called *Bayesian* approach has sometimes been accused of applying the rigorous machinery of probability theory to inputs which may be guesswork or supposition.

**Web link:** `www.dcs.qmul.ac.uk/~norman/papers/probability_puzzles/cancer.html`

**Further reading:** *Making Decisions, 2nd Ed.* by D.V. Lindley, John Wiley & Sons, 1985.